JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 24, No. 1, January–February 2001

Resolution of Conflicts Involving Many Aircraft via Semidefinite Programming

E. Frazzoli,* Z.-H. Mao,† J.-H. Oh,‡ and E. Feron§
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Aircraft conflict detection and resolution is currently attracting the interest of many air transportation service providers and is concerned with the following question: Given a set of airborne aircraft and their intended trajectories, what control strategy should be followed by the pilots and the air traffic service provider to prevent the aircraft from coming too close to each other? This paper addresses this problem by presenting a resolution methodology whereby each aircraft proposes its desired heading while a centralized air traffic control authority resolves any conflict arising between aircraft, while minimizing the deviation between desired and conflict-free heading for each aircraft. The resolution methodology relies on a combination of convex programming and randomized searches: It is shown that a version of the planar, multiaircraft conflict resolution problem, accounting for all possible crossing patterns among aircraft, might be recast as a nonconvex, quadratically constrained quadratic program. For this type of problem, there exist efficient numerical relaxations, based on semidefinite programming, that provide lower bounds on the best achievable objective. These relaxations also lead to a random search technique to compute feasible, locally optimal, and conflict-free strategies. This approach is demonstrated on numerical examples and discussed.

I. Introduction

THE air transportation system is currently the object of extensive research, following the sustained growth of air traffic over the past many years. The current en route air traffic control system consists of the following elements:

1) There is a geographical network whose nodes are navigation beacons (VHF omnidirectional range and distance measuring equipment systems), whose links are air routes. The aircraft are allowed to fly only along these routes (with possible exceptions at those altitudes where air traffic density is very low). Flying on segments connecting two navigation beacons makes the problem of aircraft navigation and automated guidance particularly easy, although recent accidents seem to have been caused by apparent ambiguities about the identity of navigation beacons.

2) Approximately 1500 en route air traffic controllers regulate the aircraft flow across this network and make sure no hazardous situation develops, whereby two aircraft might collide with each other (aircraft conflicts). The network structure of the aircraft routing system allows the air traffic control specialist to characterize a priori aircraft conflict geometries and their location during nominal operations: Conflicts are usually located at the nodes of the network. Knowing the conflict location a priori enables the decomposition of the airspace into sectors, managed by individual air traffic controllers. The boundaries of sectors are located away from the network nodes and therefore away from the most common conflict locations.

Many decades of working experience have demonstrated that this network-based architecture is safe. However, it suffers from strong perceived drawbacks, such as systematic indirect routing between origin and destination, and in general a lack of freedom for aircraft pilots. The advent of a new generation of Global Navigation Satellite System, in particular GPS, has removed in principle the limitations of the ground-based navigation infrastructure. In particular, it is now very easy to obtain precise aircraft position anywhere over the United States and not only on a predetermined set of routes (although this idea, also named area navigation, has been demonstrated to be feasible using the conventional navigation infrastructure, provided aircraft are equipped with additional computational devices). As a consequence, operational concepts such as Free Flight[1] have been proposed by airlines and the Federal Aviation Administration to remove the routing constraints imposed by the conventional, fixed-route system. Under Free Flight each aircraft would be able to optimize its trajectory according to several factors such as perceived safety, weather, direct operating costs, and coordination with other flights.[2] However, the safety of a new concept of operations that sharply departs from the current, network-based architecture remains to be proven. In particular, the lack of predictability of conflict locations under Free Flight seems to increase the apparent complexity of conflict detection and resolution for the human operator. This issue is currently under study. In addition, the set of standards over which operational concepts are evaluated has evolved from empirical evaluation decades ago to a sophisticated and very difficult certification process, which makes any new concept of operations very challenging to implement. Thus, Free Flight offers a wide array of new challenges to the research community.

This paper considers the problem of resolving conflicts arising between airborne aircraft, while accounting for aircraft heading and speed preferences. Conflicts involving several (more than two) aircraft will be considered for the following reasons: First, conflicts involving a pair of aircraft have been the object of numerous studies already.[3–7] Second, conflicts involving more than two aircraft have been shown to occur in high-density sectors[8,9]: Although more than two aircraft are rarely directly in conflict with each other, indirect conflict is a distinct possibility, whereby the solution to the conflict involving one pair of aircraft generates a secondary conflict with a third, neighboring aircraft. Several other approaches consider conflicts involving multiple aircraft.[10–12] A comprehensive review has recently appeared.[13]

The current air traffic control operations are based on conflict avoidance rules and controller experience. Rule-based approaches might work for the case involving two aircraft,[4] but may require a prohibitive number of rules to handle all situations arising when more than two aircraft are involved. The present paper concentrates on optimization-based approaches, which avoid the explicit elicitation of rules. This computational approach follows the spirit of

Received 25 August 1999; revision received 23 March 2000; accepted for publication 31 March 2000.

*Research Assistant, Laboratory for Information and Decision Systems, Department of Aeronautics and Astronautics, Room 35-223, 77 Massachusetts Avenue. Member AIAA.

†Research Assistant, Laboratory for Information and Decision Systems, Department of Aeronautics and Astronautics, Room 35-223, 77 Massachusetts Avenue.

‡Postdoctoral Associate, Department of Mechanical Engineering, Room 35-223, 77 Massachusetts Avenue.

§Associate Professor, Laboratory for Information and Decision Systems, International Center for Air Transportation, Department of Aeronautics and Astronautics, Room 33-417, 77 Massachusetts Avenue. Senior Member AIAA.

previous authors: Niedringhaus[14] proposed linear programming as a convenient modeling framework to formulate and efficiently solve conflicts arising among several aircraft. Durand et al.[8] consider the same problem and propose to use genetic algorithms and linear programming to determine optimal maneuvers to solve conflicts arising among multiple aircraft. Although both approaches emphasize (but are not limited to) planar conflict problems, the latter approach differs from the former in that it also optimizes the conflict resolution maneuver over possible crossing patterns, whereas the former approach requires a priori knowledge of the crossing pattern among aircraft.

In this paper we will present an approach to the problem that is both computationally efficient and solidly rooted in recent advances in convex optimization to solve highly nonconvex, possibly combinatorial optimization problems[15–17]: We formulate the planar conflict resolution problem as a nonconvex, quadratically constrained quadratic program. This problem is then approximated by a convex, semidefinite program, for which very efficient solutions exist. The optimal solution to this convex program is then used to randomly generate feasible and locally optimal conflict resolution maneuvers. Based on this algorithm, we then outline a distributed conflict management architecture whereby individual aircraft are able to express their heading and speed preferences at regular time intervals and are always given conflict-free, straight paths. The ability to optimize conflict resolution over all crossing patterns not only leads to better solutions, but also allows us to identify the best crossing pattern rules, which can be used as the basis of subsequent conflict resolution queries.

The paper is organized as follows: First, a simple model of air traffic is introduced; the basic conflict avoidance problem is then formulated using that model, and an initial resolution methodology is proposed. Second, the combinatorial aspects of the conflict avoidance problem are discussed. The problem is formulated as a nonconvex, quadratically constrained program; an approximate method to solve this program is introduced, based on a combination of convex programming with randomization schemes. Last, numerical examples and comparisons are presented and discussed. A symmetric conflict involving eight aircraft is first introduced, and it is shown that the best solution to that problem is not symmetrical (roundabout-type conflict resolution pattern). Then a conflict involving two streams of aircraft is discussed and solved. The resulting conflict resolution strategies are compared with those proposed in the existing literature.

II. Air Traffic Models and Problem Formulation

A. General Considerations

Like many problems of automatic control and operations research, the most challenging issue when dealing with problems in air traffic control appears during the modeling phase, that is, the boundaries of the system under study are not always very well identified.

In this paper we are interested in solving conflicts arising among several aircraft. For that purpose we assume that a finite set of aircraft has been isolated from the rest of the air transportation system. Several criteria can be used to detect those aircraft simultaneously involved in the same conflict.[3,6,8]

Although designing and analyzing systems for aircraft conflict detection and resolution needs to account for three dimensions, this paper will investigate air traffic evolving in two dimensions (planar conflict resolution): all aircraft are assumed to evolve in the plane. This paper can be extended to three dimensions, at the expense of more complicated notation. However, while vertical maneuvers appear to be most efficient for tactical conflict resolution (such as in the case of Traffic Alert and Collision Avoidance System), horizontal maneuvers might be more suitable for the strategic conflict resolution context considered in this paper because they induce less passenger discomfort, do not require flight level changes, and thus may not perturb the vertically stratified traffic structure as it exists today in the en route airspace.

Following a model first introduced and justified by Andrews,[18] we assume the state of each aircraft to be described by its position and its speed. We assume that, for a given conflict scenario, resolution maneuvers consist of simultaneous and instantaneous speed and bearing changes for all aircraft involved in the conflict. Although unrealistic, this assumption is acceptable at the temporal and spatial scales under consideration (conflicts are usually considered and solved about 10 min ahead). The proposed methodology would entail a centralized conflict resolution architecture. However, the pilots are free to indicate their desired velocities and headings. Thus the overall conflict resolution scheme is a mix of a centralized decision making structure for safety and decentralized preferences for efficiency. Roughly speaking, the same structure is currently adopted in air transportation architectures involving cooperation between the air traffic control service provider and the users.

B. Notations

Let n be the number of aircraft involved in one conflict, and let each aircraft be identified by its index $i \in \{1, \ldots, n\}$. Denote the initial position of aircraft i by $p_{i,0}$ and its initial velocity by $v_{i,0}$. Denote its position at any time t by $p_i(t)$ (or the shorthand p_i). Denote the commanded velocity changes by u_i. We will use a double-index notation for aircraft relative positions and velocities. Thus, the relative position p_{ij} is defined by $p_{ij} = p_i - p_j$; the relative speed v_{ij} is defined as $v_{ij} = v_i - v_j$, and the relative velocity changes will be noted $u_{ij} = u_i - u_j$.

C. Collision Avoidance Constraints

Conflict resolution constraints can be expressed in many ways. Although expressing collision avoidance constraints in terms of a given minimum miss distance appears to be the most attractive option from a geometrical standpoint, it can be better substituted for a time-based separation criterion, especially when considering tactical conflict resolution. The present context is concerned with strategic conflict resolution. In this case a distance-based criterion is acceptable because the main factor for this separation requirement is radar resolution. Assume then 1) a minimum safety distance d_s, 2) no initial conflict between aircraft, and 3) that aircraft follow straight trajectories at constant speed. The conflict avoidance constraint is then shown graphically in Fig. 1 for a given aircraft pair (i, j) and can be written as

$$p_{0ij}^T(v_{0ij} + u_{ij}) + \|v_{0ij} + u_{ij}\|\sqrt{\|p_{0ij}\|^2 - d_s^2} \geq 0 \qquad (1)$$

for each aircraft pair (i, j), where $\|\cdot\|$ is the Euclidean norm. For each aircraft pair feasible solution sets under the conflict avoidance constraint are represented by the union of the half planes defined by the two linear constraints

$$(v_{0ij} + u_{ij})^T n_{1ij} \geq 0 \qquad (2)$$

and

$$(v_{0ij} + u_{ij})^T n_{2ij} \geq 0 \qquad (3)$$

where n_{1ij} and n_{2ij} are shown in Fig. 1.

D. Maneuvering Constraints

The maneuvering constraints of an en route aircraft are significant. In particular, while en route at high altitude, the speed range of an aircraft is narrow. Moreover, passenger comfort and trajectory smoothness requirements can be translated into constraints on bearing changes. In this paper we will follow the formulation proposed by Niedringhaus,[14] where speed changes are constrained to stay within a given set around the current aircraft speed: The set of possible changes is the convex set of possible speed commands shown in Fig. 1 (right) and is mathematically described by the following quadratic and linear constraints:

$$\|v_{0i} + u_i\| \leq v_{\max}, \qquad (v_{0i} + u_i)^T v_{0i} \big/ \|v_{0i}\| \geq v_{\min} \qquad (4)$$

Usually, $(v_{\max} - v_{\min})/v_{\max} \leq 0.1$ for most commercial jet aircraft at high altitudes. At lower speeds the aircraft encounters stall buffeting. At higher speeds the aircraft encounters Mach buffeting. At lower altitudes the speed range can increase considerably.

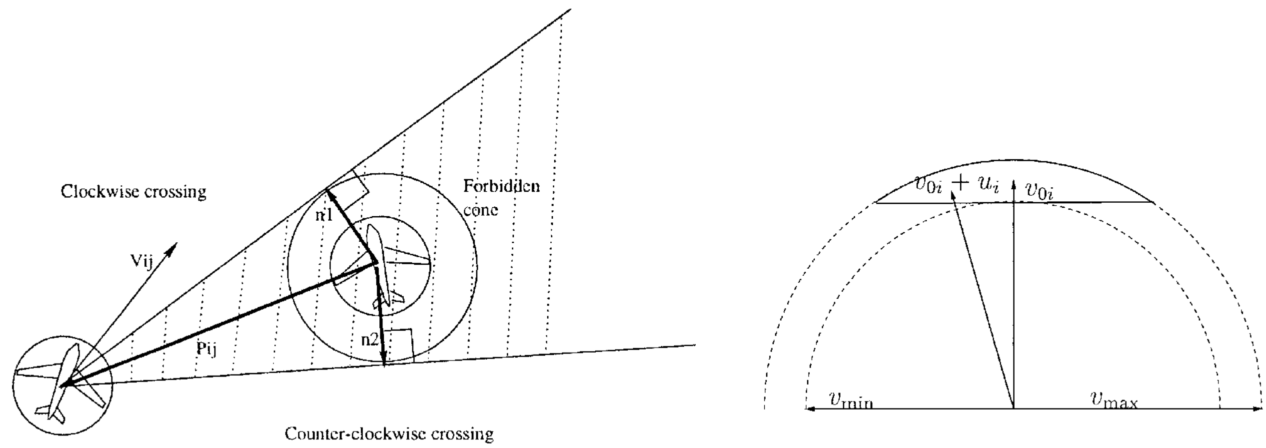

Fig. 1 Constraints on aircraft maneuvers: left, conflict avoidance constraints; right, maneuvering constraints.

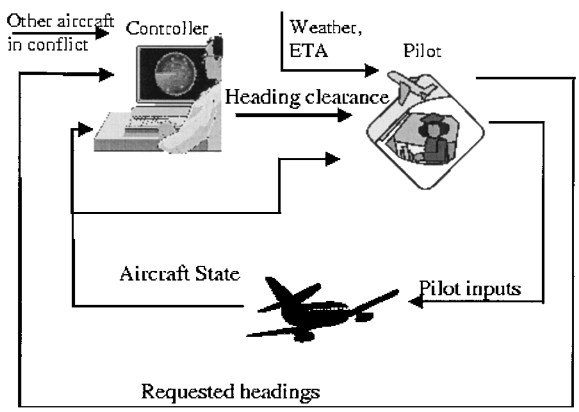

Fig. 2 Mixed centralized/decentralized air traffic control scheme.

E. Cost Function

The cost function is chosen so as to minimize the speed deviations from the desired speed as expressed by each aircraft. It is chosen to be a quadratic function of the speed deviations

$$J = \sum_{i=1}^{n} \|u_i - u_{i,d}\|^2 \tag{5}$$

which is a measure of the total energy necessary for conflict avoidance. In this context $u_{i,d}$ is the desired speed deviation. Choosing quadratic objective functions is a relatively standard practice,[19] but may be replaced by other convex objective functions.[14] The cost function (5) may also incorporate weighting terms, for example, to emphasize the cost of longitudinal speed changes over the cost of lateral speed changes.

F. Control Methodology for Conflict Avoidance

The proposed control methodology will comprise two loops, as shown in Fig. 2. The first loop manages the conflict detection and resolution and provides control commands so that straight aircraft trajectories are conflict free over a given time horizon (in the case of the aircraft clusters considered in this paper, this horizon is ∞). In general, it is considered that 10 to 20 min is an optimal horizon that trades off between deviation cost and uncertainty of conflict prediction.[6,8] The second guidance loop provides speed vector preferences at a higher rate than the chosen conflict-free horizon (for example 5 min). Although the operation of the first loop is done by a centralized algorithm, the second loop can be either centralized or decentralized. In the former case the desired speed vector for each aircraft is set by the air traffic controller, whereas in the latter case, each aircraft chooses its preferred course, such as in the case of Free Flight.[2] The pilot may then express route preferences according to other safety or economic criteria, such as weather or expected arrival time constraints.

Compared with other strategies, this strategy offers the following characteristics: first, the centralized conflict avoidance loop only computes conflict-free, straight trajectories. Although this approach is obviously motivated first by computational requirements (as described later), it also offers an attractive option to pilots and controllers alike. Indeed, segmented trajectories require significant attention on the part of the pilot and the controller if they are not totally automated and are therefore subject to pilot lack of attention and possibly pilot maneuvering delays.[20] Thus, although many existing approaches propose segmented, conflict free trajectories with little or no guarantees about what happens if way points are missed, the proposed approach always generates straight, conflict-free trajectories over a time horizon longer than that necessary and generates segmented trajectories only through updates from individual preferences.

III. Conflict Resolution Loop

A. Problem Properties

The main feature of the conflict resolution problem presented in the preceding paragraphs is its inherent combinatorial nature. The complexity of this problem seems to grow exponentially with the number of aircraft involved in the conflict. This may be seen using the following intuitive argument: The number of aircraft pairs involved in the solution to one conflict involving n aircraft is $n(n-1)/2$. For each aircraft pair the conflict resolution algorithm needs to decide whether each crossing pattern corresponding to each aircraft pair should be clockwise (the vector p_{ij} rotates clockwise) or counterclockwise (the vector p_{ij} rotates counterclockwise). Once the crossing pattern is chosen, then the conflict resolution problem becomes a convex, quadratic optimization problem (Other problem formulations have led to alternative convex optimization problems, such as linear programs.[8,14]). Solving convex, quadratic programs is particularly simple, and their theoretical computational complexity has recently been shown to be polynomial.[15]

Thus, much of the complexity in the proposed conflict resolution formulation is to find an optimum crossing pattern. In this section we propose to investigate and demonstrate via numerical examples that quadratically constrained quadratic programming and its semidefinite relaxation can be used to achieve this goal.

B. Nonconvex, Quadratically Constrained, Quadratic Programs

The general format for a nonconvex, quadratically constrained quadratic optimization problem is the following:

Minimize:

$$z^T P_0 z + 2q_0^T z + r_0$$

Subject to:

$$z^T P_i z + 2q_i^T z + r_i \leq 0, \qquad i \in \mathcal{I} \tag{6}$$

where $\mathcal{I}$ is a given index set. In this problem the objective function and the constraints are quadratic forms. The signature of these quadratic forms is a priori arbitrary. Although this problem can be shown to be very difficult to solve in general (it includes all binary integer problems as special cases[21]), it has been the focus of recent research attention. Powerful methods, based on convex optimization, can be used to obtain approximate (but often very good) solutions, along with very good lower bounds to it. These relaxations to Problem (6) can be given a number of interpretations,[16,17,21,22] including the following: instead of looking for a specific decision variable z that solves problem (6) optimally, consider a random variable z with given first-order moment (denoted $\hat{z}$) and second-order moment denoted Z (that is, $\boldsymbol{E}\, zz^T = Z$). Consider then solving problem (6) on average over that distribution, that is, the average value of the cost over the variable z is minimized, subject to the requirement that the average value of the left-hand side of the constraints in Eq. (6) be less than zero. It is easy to see that this relaxed problem can be written as follows:

Minimize:

$$\boldsymbol{Tr}\, P_0 Z + 2q_0^T \hat{z} + r_0$$

Subject to:

$$\boldsymbol{Tr}\, P_i Z + 2q_i^T \hat{z} + r_i \leq 0, \qquad i \in \mathcal{I}$$

$$\begin{pmatrix} Z & \hat{z} \\ \hat{z}^T & 1 \end{pmatrix} \geq 0 \tag{7}$$

The last constraint is added to ensure that the covariance matrix $Z - \hat{z}\hat{z}^T$ is positive semidefinite. In addition to providing lower bounds (which may be interpreted as limits of performance to the original, nonconvex problem), the random distribution defined by $\hat{z}$ and Z can also be used to obtain good feasible solutions to the original problem by searching randomly across such a distribution, thus proposing one form of efficient randomized algorithm.[23] If $Z = \hat{z}\hat{z}^T$, then the distribution in fact consists of a unique point, and in this case this point is then the optimal solution to problem (6) as well. Cases where this is known to occur systematically include the case when $\mathcal{I}$ contains only one element (presence of a single quadratic constraint[24]). Nontrivial cases where this relaxation has been known to work very well include the work by Goemans and Williamson[16] and the work by Karger et al.[25] In both cases the semidefinite relaxation was followed by an algorithm examining several random draws from the distribution defined by $(\hat{z}, Z)$. An interesting feature of the proposed relaxation is that, although it appears to approximate the original, nonconvex problem very well, it also can be solved in polynomial time, thus yielding interesting perspectives for real-time applications.[17,26]

Previous applications of this approach include the solution to a variety of problems appearing in robust control systems analysis,[22,27] analysis via linear matrix inequalities,[17] actuator placement problems,[28] network optimization problems, semiconductor manufacturing and quantum physics, as well as communications.[29] It includes in particular all linear integer programming problems as special cases.

The problem that remains is then to find what strategies should be chosen to eventually find good, feasible solutions to the problem. The latter issue has been dealt with in many different fashions in the past: it often happens that no randomized solution is feasible, yet custom-designed algorithms have been able to retrieve very good solutions[16] from these initial random solutions. One strategy is the following: Considering the problem (6), one can build a conservative approximation of it by keeping all convex constraints unchanged and linearizing the nonconvex constraints in the vicinity of the random sample, a standard practice in nonlinear optimization theory and practice.[15,21]

C. Formulation of the Conflict Avoidance Problem and Solution Procedure

Much of the research involved in the solution to the planar conflict resolution problem hinges on the ability to formulate usable quadratic constraints. (There is a thorough treatment of this issue in the case of linear integer programming.[30]) For the current problem aircraft maneuvering constraints (4) and objective function (5) are already expressed directly as (convex) linear or quadratic expressions on the aircraft's speed vectors. The conflict avoidance constraint (1) can be written as a quadratic constraint by the following proposition:

Proposition 3.1: The constraint (1) is equivalent to the set of quadratic constraints

$$p_{0ij}^T(v_{0ij} + u_{ij}) + w_{ij}\sqrt{\|p_{0ij}\|^2 - d_s^2} \geq 0, \qquad \|v_{0ij} + u_{ij}\|^2 \geq w_{ij}^2 \tag{8}$$

where $w_{ij} \geq 0$ are new slack variables.

This proposition is trivial to prove.

Constraints like Eq. (1) are traditionally transformed into mixed integer linear constraints using standard methods.[31] The proposed method is an attractive and efficient alternative to these traditional approaches.

The preceding constraints and cost function form a nonconvex quadratic program of the form (6), with $z := [u_1, u_2, \ldots, u_n, w_{12}, \ldots, w_{(n-1),n}]^T$.

If the optimal solution Z to the semidefinite relaxation has unit rank, then $\hat{z}$ is the solution to the original problem. Otherwise, the following randomization procedure is applied: considering the Gaussian distribution with mean $\hat{z}$ and covariance $Z - \hat{z}\hat{z}^T$, pick samples $\tilde{z}$ according to that distribution. The linearization procedure is then to pick the crossing pattern for each aircraft pair by computing

$$C = \text{sign}[p_{0ij} \times (v_{0ij} + \tilde{u}_{ij})]$$

where $\tilde{u}_{ij} = \tilde{u}_i - \tilde{u}_j$ is computed from $\tilde{z}$ and $\times$ is the usual outer product between two planar vectors. The crossing pattern is then chosen to be counterclockwise if $C = 1$ [the linear constraint (2) is chosen] and clockwise if $C = -1$ [the linear constraint (3) is chosen]. By convention, we will assume that the crossing pattern is clockwise in the very unlikely case when $C = 0$.

Once the crossing pattern is chosen, the corresponding convex optimization problem can be solved using recently introduced optimization methods[15] or otherwise.

IV. Numerical Examples

The proposed approach is now illustrated on a number of numerical experiments. The scenarios for the examples have been selected to test the performance of the proposed methodology on purposefully difficult problems. Although very unlikely to occur in real world operations, the selected scenarios can be used to show the effectiveness of the methodology in a stressful situation and to gain some insight on the provided solutions. First, a Free-Flight-like scenario is presented and solved using the proposed numerical approach. Then, an unrealistic but geometrically elegant symmetric conflict involving eight aircraft is considered. The optimal solution to this problem is not a symmetric turnaround. Then, an example where two aircraft streams fly miles-in-trail is considered. The proposed approach works better than approaches that choose aircraft crossing patterns a priori. Branch-and-bound tests reveal that the proposed procedure produces excellent solutions (close to optimal) for these two examples. In addition, it is shown how the optimization algorithm automatically generates conflict avoidance maneuvers by platooning aircraft together. The optimization software SDPPACK[32] was used for the numerical experiments.

A. Random Encounter Pattern

A set of 10 aircraft flying at a nominal, desired speed of 200 kn was positioned and oriented at random, as shown in Fig. 3. This initial configuration generates eight conflicts among those aircraft. The conflicts need to be solved simultaneously because of the generally convergent nature of the aircraft flow. In this example only one resolution command is issued to all aircraft. The maximum speed for all aircraft is $v_{\max} = 220$ kn, and the minimum speed is $v_{\min} = 180$ kn. The minimum miss distance between aircraft was chosen to be $d_s = 5$ Nm in this case. In Fig. 3 and later the circles surrounding the aircraft have a diameter the size of the minimum miss distance between aircraft.

The combination of convex programming and randomized search led to a set of solutions whose best element scored a cost of 663 knt^2. The semidefinite programming relaxation yielded a lower bound on the optimal cost of 603 knt^2. Thus, the gap between the cost of the

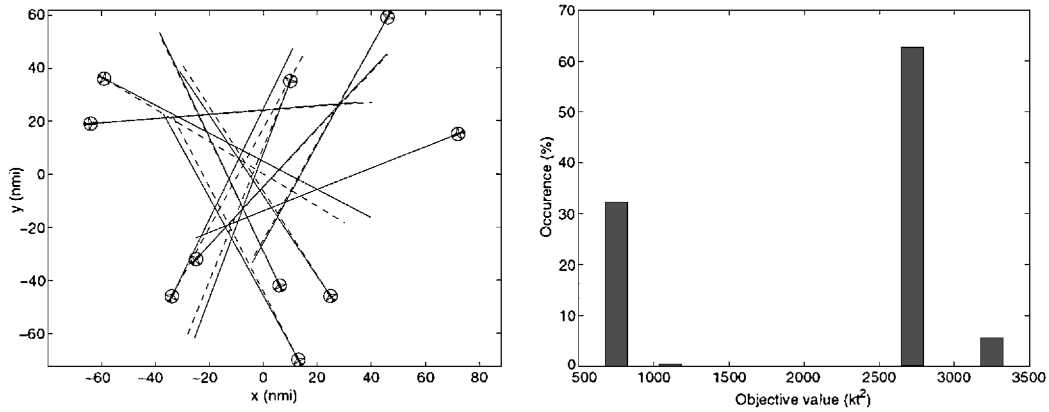

Fig. 3 Test case for multiaircraft conflict resolution algorithm: left, 10 converging aircraft; right, distribution of results from randomized algorithm; – – –, initial configuration; continuous, configuration after conflict resolution.

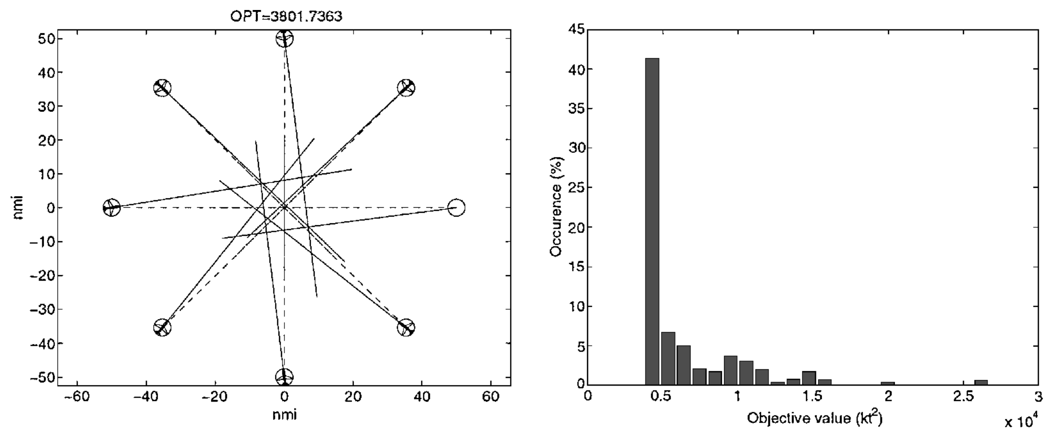

Fig. 4 Test case for multiaircraft conflict resolution algorithm: left, eight converging aircraft; right, distribution of results from randomized algorithm; – – –, initial configuration; continuous, configuration after conflict resolution.

best resolution command generated by the proposed methodology and the best possible cost is less than 10% in this case. To get an idea of the performance of the proposed algorithm in this case, we ran a simulation in which 500 random solutions were generated using the proposed approach and examined: the randomized algorithm found a feasible solution for 100% of all of the generated samples. A normalized histogram of the performance obtained for each random trial is shown in Fig. 3; it shows that for the 500 random trials sample one out of three trials yields the best found performance.

B. Symmetric Encounter Pattern

A set of eight aircraft is shown in Fig. 4. These aircraft converge to the same point at the same speed (200 kn), and it is desired to find optimal aircraft deviations so that conflicts are avoided. In this example, as in the preceding one, only one resolution command is issued to examine the performance of the proposed optimization algorithm. The maximum speed $v_{\max}$ is 220 kn, and the minimum speed $v_{\min}$ is 180 kn. An intuitive optimal solution would follow a roundabout pattern,[12] whereby every aircraft deviates its course by the same angle. The randomization algorithm found that a better solution exists, which is not symmetric and rather counterintuitive: This solution requires two airplanes to fly straight through the center (one accelerates, the other decelerates), and the others to deviate from their original course following a roundabout pattern, as shown in Fig. 4. In this figure the circles surrounding the aircraft have a diameter the size of the minimum miss distance between aircraft (5 Nm).

The best cost found is 3801.7 knt^2. The optimal cost provided by the semidefinite relaxation (and thus a lower bound on the best achievable cost) is 1100 knt^2. Thus there is a significant difference between upper and lower bounds in this case. However, the best roundabout solution corresponds to a cost of 5486 knt^2, and thus the best found solution represents a 40% improvement compared with the roundabout solution. Any rotation or flip to that solution remains valid as well. Again, we ran a simulation in which 500 trial solutions were generated and examined: the randomized algorithm found a feasible solution for about 68% of all of the generated samples, and the distribution of solutions according to performance is very skewed toward the best cost found, as may be seen in Fig. 4. Thus, it usually takes only a few random trials to generate a good solution. We also compared the performance of the proposed algorithm with that of a purely random scheme to generate crossing patterns: in the purely random case less than 0.5% of the generated crossing patterns yielded feasible solutions, and the generated solutions were always considerably worse in terms of performance than those generated via the proposed approach.

Because the gap between the upper and lower bounds is large in this example, the algorithm does not provide a clear indication of how close the best computed feasible solution is to the optimal solution. To obtain more information about the optimality of the solution provided by the proposed methodology, a straightforward branch-and-bound algorithm was implemented. Branching was implemented by choosing crossing patterns for each aircraft pair in ascending order (the aircraft pairs are assumed to have been organized in an ordered list), and bounding was achieved through the application of the proposed semidefinite relaxation algorithm (which provides lower bounds) and the randomization procedure (which provides upper bounds) to the nonbranched crossing patterns for the remaining aircraft pairs. Details about branch-and-bound algorithms can be found in most optimization textbooks.[31] Although considerably more time consuming, branch-and-bound procedures are global optimization methods. In this case the branch-and-bound procedure showed that the global optimum value for this problem is 3673 knt^2, thus within 4% of the value found with the initial relaxation approach (3801.7 knt^2).

C. Crossing Aircraft Streams

This example illustrates the performance of the proposed algorithm when handling the intersection between two aircraft streams. Indeed, a classical solution to that problem would rely on fixed aircraft routing and sequencing aircraft at the intersection, which would result in larger than necessary separation between aircraft in the same stream. This example was inspired from the article by Niedringhaus.[14] In that article the author used a linear programming approach to solve this problem and used a fixed and predetermined crossing pattern. The example below shows that some improvements and insight can be obtained by considering the option to also optimize over the crossing patterns as well. Considering first the case when the aircraft are allowed to perform one and only one simultaneous and instantaneous turn, this section then considers the case when the aircraft are issued several resolution commands, in order to recover their initial course.

1. *Single Turn Pattern*

Two strings of four aircraft spaced 25 miles-in-trail are converging toward each other as shown in Fig. 5. The aircraft's speeds are 200 kn, $v_{\min} = 180$ kn, and $v_{\max} = 220$ kn. The minimum miss distance was chosen arbitrarily to be 20 Nm. As a result, it is impossible to resolve conflicts arising between these two aircraft streams

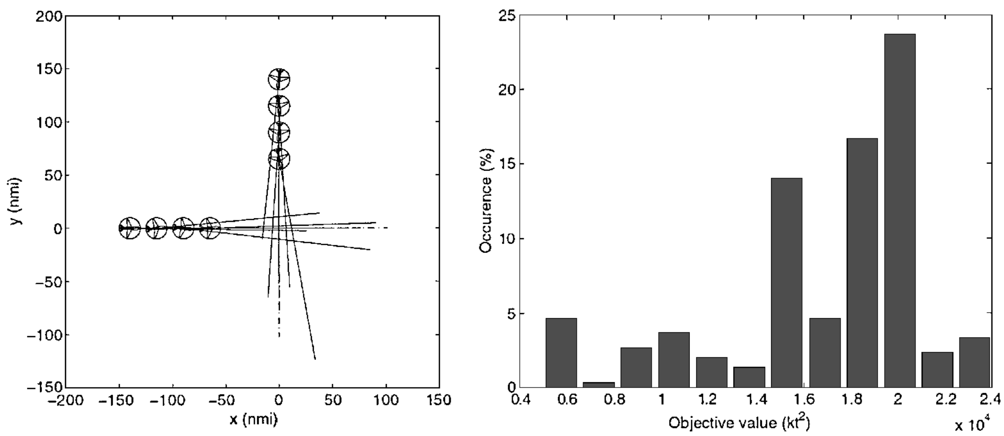

Fig. 5 Test case for multiaircraft conflict resolution algorithm: left, two converging lines of four aircraft; right, distribution of results from randomized algorithm; – – –, initial configuration; continuous, configuration after conflict resolution.

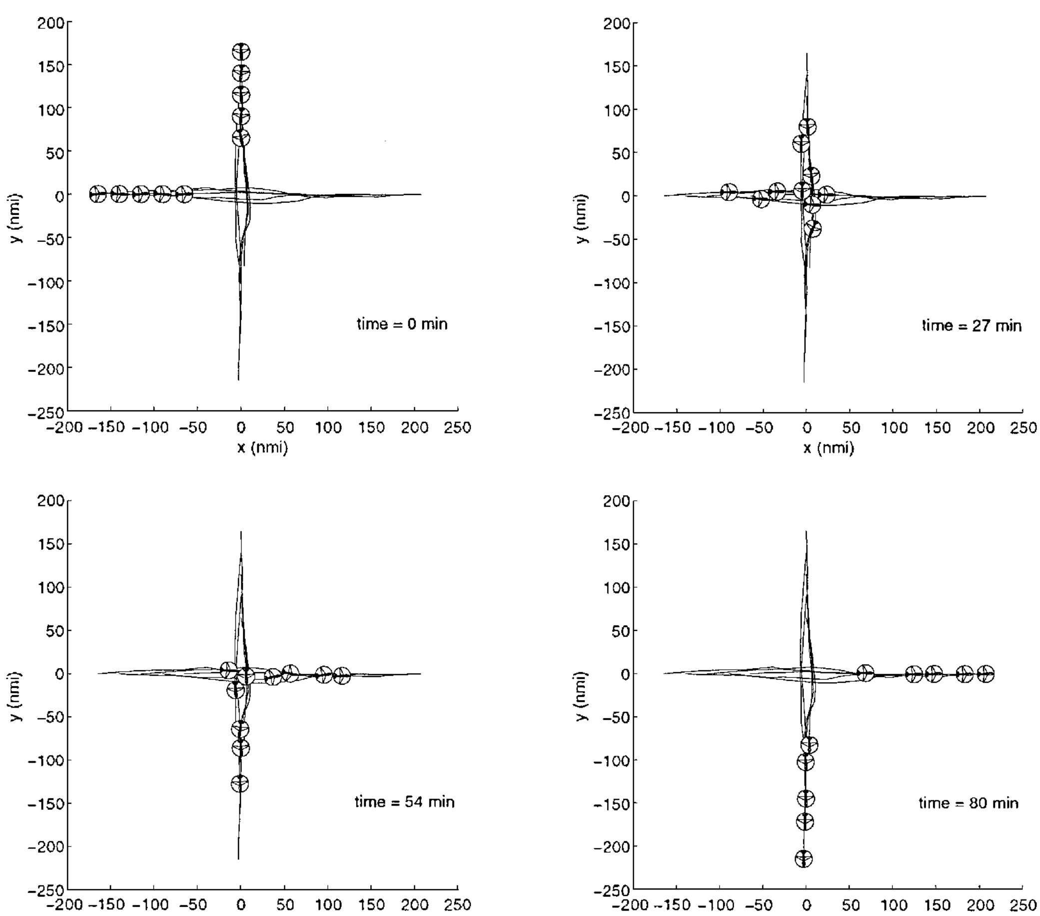

Fig. 6 Two crossing aircraft strings; platooning is the resulting behavior to solve the conflicts arising between the two aircraft strings.

by using staging without path deviations: the distance between two consecutive aircraft in one stream does not allow the controller to insert an aircraft from the other stream between them without creating a conflict. The results of the proposed approach are illustrated in Fig. 5. Again, the outcome of the semidefinite relaxation generates a probability distribution. About 80% of the 500 randomly generated crossing patterns resulted in feasible conflict resolution maneuvers, whereas only 1% of the crossing patterns generated from a uniform distribution over all possible crossing patterns resulted in feasible conflict resolution maneuvers. The best feasible solution found generated a cost of 4968.3 knt^2, whereas the semidefinite relaxation provided a lower bound value of 3888.6 knt^2. A subsequent, costlier branch-and-bound optimization revealed that the optimum value for this problem is 4959 knt^2. Thus the best solution value found with the proposed procedure is within less than 0.2% of optimality. Compared with the published solutions,[14] the proposed optimization procedure results in smaller trajectory deviations because it optimizes over the crossing patterns as well.

2. Multiple Turn Simulation

Two strings of five aircraft each are converging toward each other. In this section the proposed resolution procedure was used every 5 min to update the aircraft trajectories and possibly allow the aircraft to recover their nominal flight paths. Thus, every 5 min, a new, conflict-free rectilinear trajectory is generated according to the scheme shown in Fig. 2, and this trajectory is conflict free for the aircraft set under consideration. The guidance law that drives the preference of each aircraft is a simple proportional guidance law, whereby the desired speed vector is proportional to the lateral deviation of the aircraft from its intended (rectilinear) course. In this case the guidance law is chosen so that if an aircraft is granted its desired speed vector, it returns to its desired course within one time step (5 min). The velocity and heading constraints of all aircraft are the same as in the preceding examples.

Figure 6 shows the trajectories followed by the 10 aircraft, along with four snapshots taken at $t = 0$, 27, 54, and 80 min. In this case the chosen strategy is platooning: because the spacing between aircraft does not allow the two aircraft streams to cross by staging

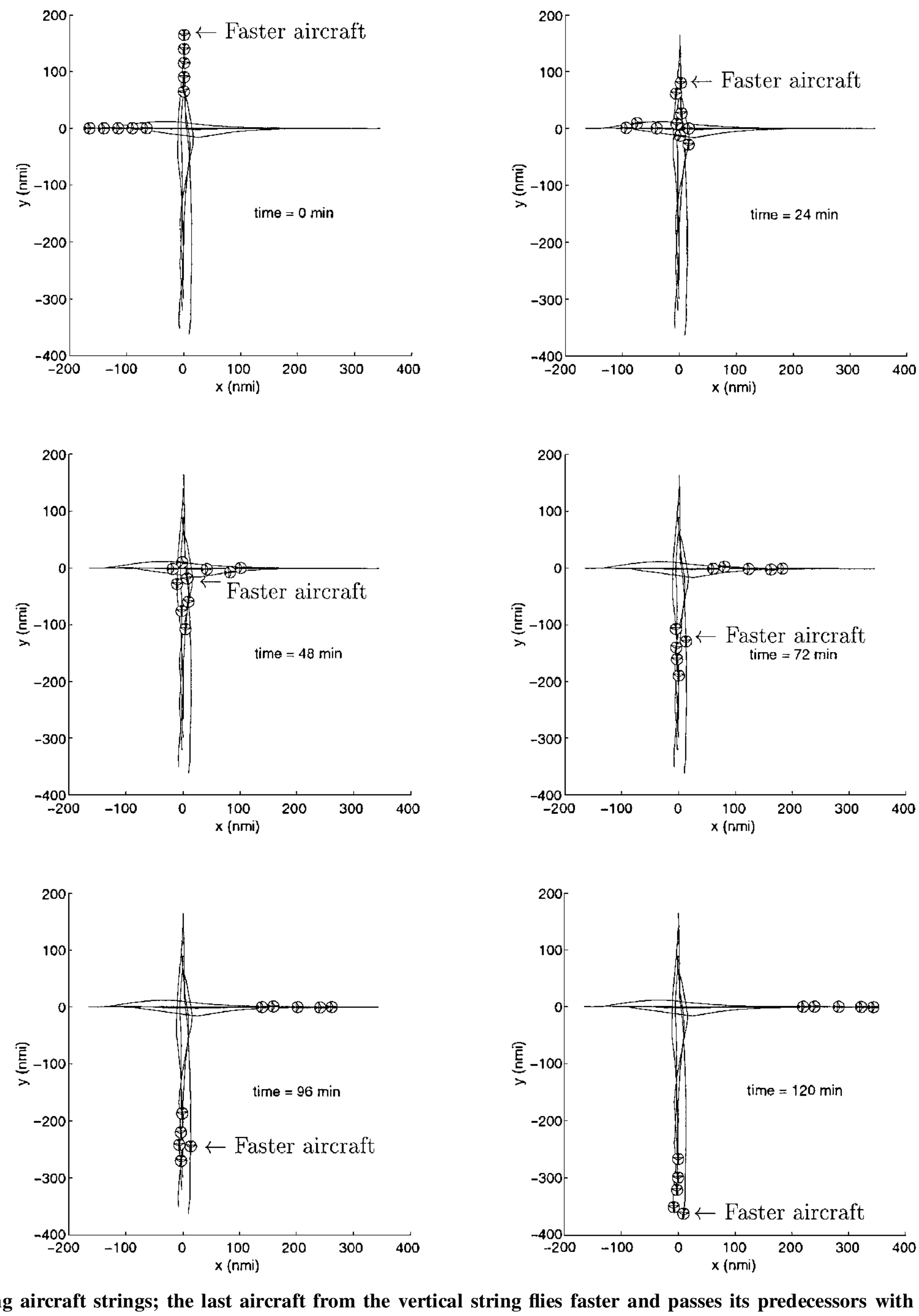

Fig. 7 Two crossing aircraft strings; the last aircraft from the vertical string flies faster and passes its predecessors with simultaneous conflict resolution.

aircraft without generating conflicts, the proposed algorithm groups aircraft in pairs. Interestingly enough, platooning has been proposed as a viable, although heuristic option in many intelligent, hierarchical transportation systems.[33] After the conflict is resolved, the aircraft recover their positions and trail each other again. This example shows another interesting aspect of the proposed optimization-based approach to solve conflict arising among aircraft: it allows the engineer to find and/or justify specific hierarchical structures meant to reduce the complexity of systems involving many interacting vehicles.[33]

3. *Multisegmented Simulation with Aircraft Passing*

Next, the following scenario was simulated: considering again two intersecting aircraft streams, it is assumed that one aircraft wants to fly faster than the other aircraft. Initially, all aircraft fly at the same speed (200 kn); however, the accelerating aircraft progressively indicates a faster desired speed (up to 300 kn). Figure 7 shows a simulation of the aircraft flow at $t = 0$, 24, 48, 72, 96, and 120 min. Again, Fig. 7 shows that all conflicts are avoided, while the faster aircaft is allowed to pass the preceding aircraft.

V. Conclusion

In this paper the problem of resolving conflicts arising among several aircraft has been considered. A conflict resolution methodology combining decentralized aircraft preferences with centralized conflict resolution while minimizing path deviations from the desired paths is proposed. The centralized conflict resolution system is based on the formulation of a related nonconvex quadratic programming problem and its solution via semidefinite programming combined with a randomization scheme. Numerical simulations have indicated that the proposed approach can deal efficiently with complex conflict scenarios involving multiple aircraft.

The use of computationally efficient algorithms throughout the conflict resolution process could make the methodology applicable to on-line implementation. However, the randomized nature of the algorithm raises serious concerns about the ability to certify such a method in a purely deterministic framework.

Alternatively, the proposed method could be used in a simulation environment to analyze specific conflict resolution problems and determine optimal solutions. These solutions could then be used as limits of performance benchmarks and compared with suboptimal, decentralized solutions. They can also be used to uncover and justify specific aircraft flow patterns (such as aircraft platooning) that may be used in future, rule-based conflict resolution systems.

Acknowledgments

The authors would like to thank J. Marc Shewchun for his exploratory investigations. This work was supported by NASA Ames Research Center, under Grant NCC2-1044.

References

[1]"RTCA Task Force 3: Free Flight Implementation, Final Report," Radio Technical Commision for Aeronautics, Washington, DC, Oct. 1995.

[2]Pujet, N., and Feron, E., "Flight Plan Optimization in Flexible Air Traffic Environments," AIAA Paper 96-3765, July 1996.

[3]Kuchar, J. K., and Yang, L. C., "Incorporation of Uncertain Intent Information in Conflict Detection and Resolution," *Proceedings of the IEEE Conference on Decision and Control*, 1997, pp. 1810–1815.

[4]Schild, R., "Rule Optimization for Airborne Aircraft Separation," Ph.D. Dissertation, Dept. of Econometry, Operations Research and System Theory, Technical Univ., Vienna, Dec. 1998.

[5]Kuchar, J., "A Methodology for Alerting System Performance Evaluation," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 2, 1996, pp. 438–444.

[6]Paielli, R. A., and Erzberger, H., "Conflict Probability Estimation for Free Flight," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 3, 1997, pp. 588–596.

[7]Krozel, J., and Peters, M., "Strategic Conflict Detection and Resolution for Free Flight," *Proceedings of the IEEE Conference on Decision and Control*, 1997, pp. 1822–1828.

[8]Durand, N., Alliot, J. M., and Chansou, O., "An Optimizing Conflict Solver for Air Traffic Control," *Air Traffic Control Quarterly*, Vol. 3, No. 4, 1995.

[9]Alliot, J.-M., Durand, N., and Granger, G., "FACES: a Free Flight Autonomous and Coordinate Embarked Solver," *FAA-Eurocontrol Air Traffic Management Seminar*, Orlando, FL, URL: http://www.eurocontrol.fr [cited 11 Dec. 2000].

[10]Eby, M. S., "A Self-Organizational Approach for Resolving Air Traffic Conflicts," *The Lincoln Laboratory Journal*, Vol. 2, No. 7, 1994, pp. 239–253.

[11]Zeghal, K., "Towards the Logic of an Airborne Collision Avoidance System Which Ensures Coordination with Multiple Cooperative Intruders," *Proceedings of the International Council of the Aeronautical Sciences*, AIAA, Reston, VA, 1994, pp. 2208–2218.

[12]Tomlin, C., Pappas, G., and Sastry, S., "Conflict Resolution for Air Traffic Management: A Study in Multiagent Hybrid Systems," *IEEE Transactions on Automatic Control*, Vol. 43, No. 4, 1998, pp. 509–521.

[13]Kuchar, J. K., and Yang, L. Y., "Survey of Conflict Detection and Resolution Modeling Methods," AIAA Paper 97-37001, Aug. 1997, pp. 1388–1397.

[14]Niedringhaus, W., "Stream Option Manager: Automated Integration of Aircraft Separation, Merging, Stream Management, and Other Air Traffic Control Functions," *IEEE Transactions on Man, Systems and Cybernetics*, Vol. 25, No. 4, 1995, pp. 140–147.

[15]Nesterov, Yu., and Nemirovsky, A., *Interior-Point Polynomial Methods in Convex Programming*, *Studies in and Applied Mathematics*, Vol. 13, Society for Industrial Applied Mathematics, Philadelphia, PA, 1994, pp. 11–55.

[16]Goemans, M., and Williamson, D., "Improved Approximation Algorithms for Maximum Cut and Satisfiability Problems Using Semidefinite Programming," *Journal of the Association for Computing Machinery*, Vol. 42, 1995, pp. 1115–1145.

[17]Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V., *Linear Matrix Inequalities in System and Control Theory*, *Studies in Applied Mathematics*, Vol. 15, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1994, pp. 7–35.

[18]Andrews, J. W., "A Relative Motion Analysis of Horizontal Collision Avoidance," *SAFE Journal*, Vol. 8, No. 2, 1978, pp. 7, 8.

[19]Menon, P. K., Sweriduk, G. D., and Sridhar, B., "Optimal Strategies for Free-Flight Air Traffic Conflict Resolution," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 202–211.

[20]Yang, L. C., and Kuchar, J. K, "A Prototype Conflict Alerting System for Free Flight," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 4, 1997, pp. 768–773.

[21]Vandenberghe, L., and Boyd, S., "Semidefinite Programming," *SIAM Review*, Vol. 38, No. 1, 1995, pp. 49–95.

[22]Doyle, J., "Analysis of Feedback Systems with Structured Uncertainties," *IEE Proceedings*, Vol. 129-D, No. 6, 1982, pp. 242–250.

[23]Motwani, R., and Raghavan, P., *Randomized Algorithms*, Stanford-Cambridge Program, Cambridge Univ. Press, Cambridge, England, U.K., 1997, pp. 278–305.

[24]Uhlig, F., "A Recurring Theorem About Pairs of Quadratic Forms and Extensions: A Survey," *Linear Algebra and Applications*, Vol. 25, 1979, pp. 219–237.

[25]Karger, D., Motwani, R., and Sudan, M., "Approximate Graph Coloring by Semidefinite Programming," *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, 1994, pp. 2–13.

[26]McGovern, L., and Feron, E., "Requirements and Hard Computational Bounds for Real-Time Optimization Problems," Inst. of Electrical and Electronics Engineers, New York, Dec. 1998.

[27]Safonov, M. G., "Stability Margins of Diagonally Perturbed Multivariable Feedback Systems," *IEE Proceedings*, Vol. 129-D, No. 6, 1982, pp. 251–256.

[28]Jamoom, M., Feron, E., and McConley, M., "Optimal Distributed Actuator Control Grouping Schemes," Inst. of Electrical and Electronics Engineers, Dec. 1998, pp. 1900–1905.

[29]Lovász, L., "On the Shannon Capacity of a Graph," *IEEE Transactions on Information Theory*, Vol. IT-25, No. 1, 1979, pp. 1–7.

[30]Lovász, L., and Schriver, A., "Cones of Matrices and Set-Functions and 0-1 Optimization," *SIAM Journal on Optimization*, Vol. 1, No. 2, 1991, pp. 166–190.

[31]Hillier, F. S., and Lieberman, G. J., *Introduction to Operations Research*, Computer Science Workbench, McGraw–Hill, New York, 1990, pp. 494–557.

[32]Alizadeh, F., Haeberly, J.-P. , Nayakkankuppam, M. V., Overton, M. L., and Schmieta, S., SDPPPACK Version 0.9: User's Guide, New York Univ., New York, 1997.

[33]Varaiya, P., "Smart Cars on Smart Roads: Problems of Control," *IEEE Transactions on Automatic Control*, Vol. 38, No. 2, 1003, pp. 195–207.